Exercise 1

Explain exactly what is meant by the statement that "differentiation and integration are inverse processes."

Solution

Differentiation undoes an integral, and integration undoes a derivative. For example, if one has

$$\int_{-2}^{x} f(t)\,dt,$$

the way to get $f(x)$ is to differentiate this integral.

$$\frac{d}{dx}\int_{-2}^{x} f(t)\,dt$$

$$f(x)$$

On the other hand, if one has

$$\frac{df}{dx} \qquad \text{or} \qquad f'(x),$$

the way to get $f(x)$ is to integrate this derivative.

$$\int_{0}^{x} \frac{df}{dt}\,dt \qquad \text{or} \qquad \int_{0}^{x} f'(t)\,dt$$

$$f(t)\Big|_{0}^{x} \qquad \text{or} \qquad f(t)\Big|_{0}^{x}$$

$$f(x) - f(0) \qquad \text{or} \qquad f(x) - f(0)$$

$f(0)$ is an extra constant.